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PARTICLE PRODUCTION AND SURVIVAL IN MUON ACCELERATION

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Abstract

Because of the relative immunity of muons to synchrotron radiation, the idea of using them instead of electrons as probes in high-energy physics experiments has existed for some time, but applications were limited by the short muon lifetime. The production and survival of an adequate supply of low-emittance muons will determine the available luminosity in a high-energy physics collider. In this paper the production of pions by protons, their decay to muons and the survival of muons during acceleration are studied. Based on a combination of the various efficiencies, the number of protons needed at the pion source for every muon required in the final high-energy collider is estimated.

Introduction

The relative immunity of muons to synchrotron radiation due to their large rest mass ($m_\mu = 105.7\,Mev/c^2$) suggests that they might be used in place of electrons in accelerators for high-energy physics experiments. The idea of using muons as high-energy probes has existed for some time, but applications were limited by the short muon lifetime. Muons have been used in secondary beams as "deep-inelastic" probes of hadron structure. Skrinsky[1] and others have suggested that their role could be extended by accelerating muons after production. Neuffer[2] has recently described the physics of "ionization cooling" of muons for obtaining low-emittance beams needed in high energy colliders.

The short lifetime of muons and the large emittance of initial beams will determine the essential character of any muon acceleration chain: a rapid-cycling facility, the front end of which resembles a "meson factory"[3] in which intense

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proton beams impinging on a stationary target produce copious pion beams that decay into muons for cooling and acceleration. Acceleration to the final collider energy may be in a linear accelerator or rapid-cycling synchrotron, and collisions for physics can be contemplated in either the $\mu^+\mu^-$ or μp channels. If the muons are placed in a storage ring for high-energy experiments, the useful storage time is about one muon lifetime at the collision energy or $2.197 \times 10^{-6}(E_\mu/m_\mu c^2)$ seconds. In any event the production and survival of an adequate supply of low-emittance muons will determine the available luminosity in such machines.

In this paper the production of pions, their decay to muons and the survival of muons during acceleration are studied. The survival of muons during ionization cooling is not discussed since this has been previously considered by Neuffer.[3,4] He showed that the lowest normalized emittance achievable by ionization cooling is approximately $\epsilon_N(rms) = 10^{-2}$ cm rad and is limited by multiple scattering in the absorbers. Cooling of both longitudinal and transverse emittances by a factor of 100 within one muon lifetime at the cooling energy can be achieved with an average energy gradient of about 1 MV/m, so the muon survival ratio during cooling would be $\eta_c \simeq 1/e \simeq 0.37$. This cooling efficiency can be improved by increasing the energy absorption and return per unit length in the cooler which reduces the cooling time.

In the present study a combination of the various efficiencies will suggest that for every muon required in the final high-energy collider, approximately 10^3 protons are needed in the meson factory to produce the initial pions. The basic efficiencies that control the total muon production are the pion production yield $\eta_{p\pi}$, the target efficiency η_t, the pion-muon decay efficiency $\eta_{\pi\mu}$, the cooling survival efficiency η_c and the acceleration survival efficiency η_a. Numerically $\eta_{p\pi}$ and $\eta_{\pi\mu}$ are the smallest coefficients among these and increase almost linearly with increasing momentum spread $\Delta p/p$ of the accepted beams. Significant improvement in muon production could be achieved if initial beams with momentum spreads exceeding $\pm 5\%$ could be captured prior to cooling.

Pion Production

Muons are not seen in abundance in most high energy processes unless one takes great care to detect them. The exception is charged pion decay ($\pi \rightarrow \mu + \nu_\mu$) in which muons result from essentially all decays. Pions are produced copiously from proton beams on stationary targets. In this sense muons are tertiary particles from the proton collision.

The calculated particle spectra of Grote et al[5] are useful for estimating pion production on stationary targets for primary proton momenta p_p between 12.5 GeV/c and 800 GeV/c. The spectrum $d^2N_\pi/dp_\pi\, d\Omega$ per interacting proton is relatively insensitive to the target composition. High atomic number targets are preferable however to limit depth of focus problems associated with particle production over long distances. The pion spectrum is very broad in momentum. For forward production ($\theta_\pi = 0^\circ$), the maximum in the π^- spectrum occurs at

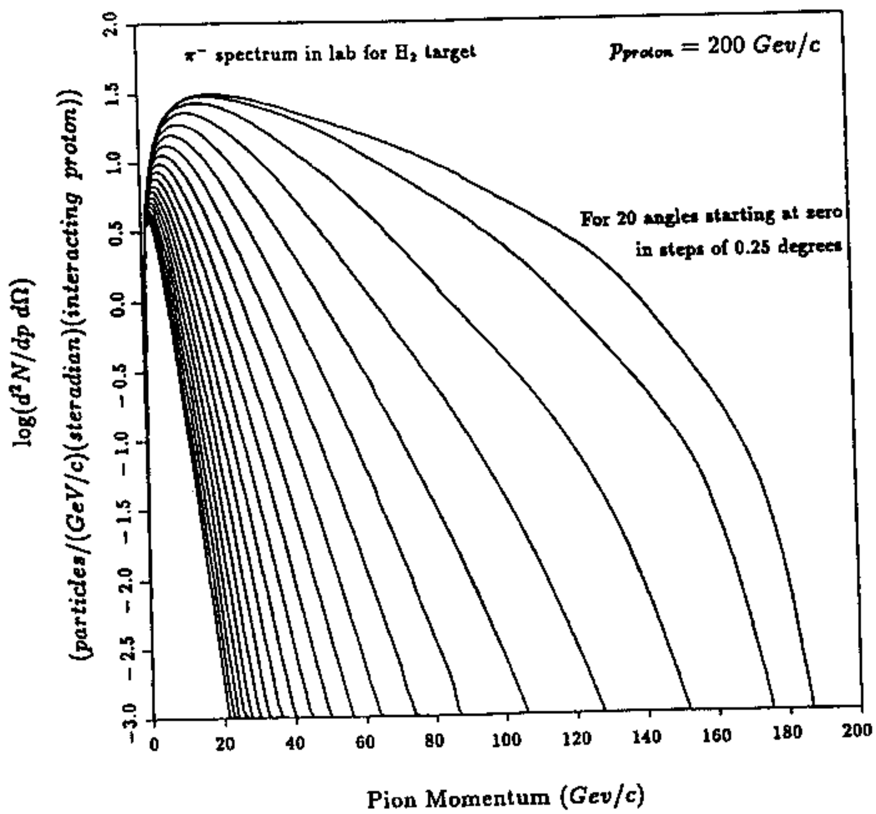

Figure 1: Negative pion spectrum from 200 GeV protons on a hydrogen target (redrawn from Ref. 5).

$p_\pi \simeq p_p/10$ with the position of the maximum decreasing with increasing angle (Figure 1). The π^+ spectrum for $\theta_\pi \simeq 0°$ is relatively flat between $p_p/10$ and $p_p/3$ but is similar to the π^- spectrum at finite angles.

Pion yields are generally increased by accepting larger production angles and momentum spreads. For most pion momenta, the differential production decreases by about an order of magnitude for angles greater than $3.5\ m_\pi c/p_\pi$ relative to forward production. This offsets the advantage of increased solid angle and limits the useful collection angle to about this value. Because of the competing effects of increasing solid angle and decreasing spectral peak at small pion momenta,the maximum pion yield obtained by integrating over angles and a given momentum spread ($\pm\Delta p/p_\pi$) occurs for $p_\pi \simeq p_p/20$. For small momentum spreads, the yield will vary almost linearly with $\Delta p/p_\pi$, but it becomes difficult to transport and capture particle bunches with momentum spreads exceeding several percent.

Tables 1 and 2 show the pion yields $\eta_{p\pi^-}$ and $\eta_{p\pi^+}$ at different proton momenta calculated from the spectra of Grote et al[5]. These yields are for a maximum accepted production angle of $\theta_{max} = 3.5\, m_\pi c/p_\pi$ and a momentum spread of $\pm 1\%$ around $p_\pi = p_p/20$. The use of an angle proportional to p_π^{-1} in this comparison has the advantage that the yields are quoted at the same normalized transverse emittance for a given beam size at the target. The yields are plotted in Figure 2 and are seen to be nearly constant for proton momenta below 100 GeV/c but then increase steadily. For a momentum spread of $\pm 5\%$, the pion yield $\eta_{p\pi}$ increases from 6% to 10% as the pion momentum increases from 5 GeV/c to 40 GeV/c.

p_p (GeV/c)	p_π (GeV/c)	θ_{max} $(mrad)$	dN_{π^-}/dp_π $((GeV/c)^{-1})$	$\eta_{p\pi^-}$ $(\Delta p/p_\pi = \pm 1\%)$
12.5	0.625	784	7.23×10^{-1}	9.04×10^{-3}
19.2	0.960	510	4.63×10^{-1}	8.88×10^{-3}
30.0	1.5	327	2.70×10^{-1}	8.10×10^{-3}
50.0	2.5	196	2.05×10^{-1}	1.02×10^{-2}
70.0	3.5	140	1.48×10^{-1}	1.04×10^{-2}
150	7.5	65.3	9.42×10^{-2}	1.41×10^{-2}
200	10	49.0	7.47×10^{-2}	1.49×10^{-2}
300	15	32.7	5.53×10^{-2}	1.66×10^{-2}
500	25	19.6	3.78×10^{-2}	1.89×10^{-2}
800	40	12.3	2.61×10^{-2}	2.08×10^{-2}

Table 1: Negative pion yields per interacting proton when $\theta_{max} = 3.5\, m_\pi c/p_\pi$.

p_p (GeV/c)	p_π (GeV/c)	θ_{max} $(mrad)$	dN_{π^+}/dp_π $((GeV/c)^{-1})$	$\eta_{p\pi^+}$ $(\Delta p/p_\pi = \pm 1\%)$
12.5	0.625	784	6.80×10^{-1}	8.50×10^{-3}
19.2	0.960	510	4.99×10^{-1}	9.58×10^{-3}
30.0	1.5	327	3.35×10^{-1}	1.01×10^{-2}
50.0	2.5	196	2.07×10^{-1}	1.04×10^{-2}
70.0	3.5	140	1.49×10^{-1}	1.05×10^{-2}
150	7.5	65.3	7.89×10^{-2}	1.18×10^{-2}
200	10	49.0	6.31×10^{-2}	1.26×10^{-2}
300	15	32.7	4.49×10^{-2}	1.35×10^{-2}
500	25	19.6	3.04×10^{-2}	1.52×10^{-2}
800	40	12.3	2.11×10^{-2}	1.69×10^{-2}

Table 2: Positive pion yields per interacting proton when $\theta_{max} = 3.5\, m_\pi c/p_\pi$.

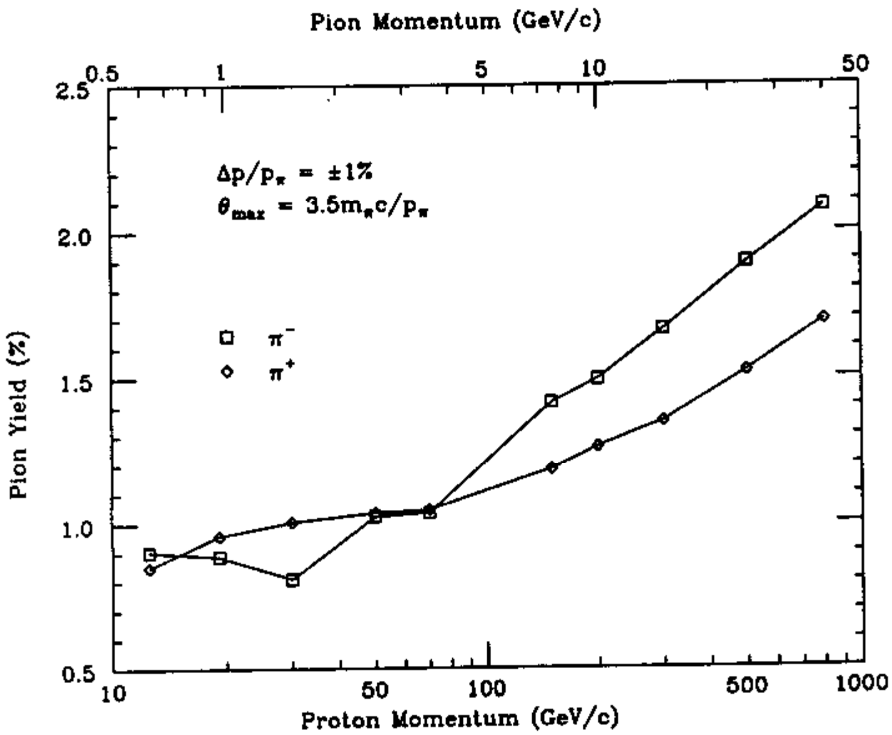

Figure 2: Pion yields per interacting proton.

The increase in pion yield above 100 GeV/c proton momentum suggests that it is advantageous to collect pions and hence decaying muons at a momentum $p_\pi \simeq p_\mu \geq 5$ GeV/c. Neuffer has shown that ionisation cooling of longitudinal emittance (energy spread) is most effective at a muon energy of about 1 Gev with little variation between 0.5 and 5 Gev because of the shape of the high-energy loss curve.[2] Longitudinal emittance cooling can be enhanced by introducing dispersion into the cooler so in fact cooling above 5 GeV is feasible.

For proton beams with momenta above 100 GeV/c, tungsten targets of length 5 to 10 cm are appropriate for secondary hadronic particle production. These lengths are comparable to the nuclear collision and interaction lengths in tungsten. The target efficiency η_t (the probability that a proton interacts producing a secondary hadronic particle which exits the target) is about 0.4 in such targets. A low emittance pion beam is desirable to reduce both the initial muon emittance and aperture requirements downstream. The proton spot size on the target should be as small as possible without causing target destruction by shock wave depletion.[6] If 95% of the proton and pion beams are within a radius $R(cm)$ at the target, the pion emittance is $\epsilon_N(95\%) \simeq 3.5\ R(cm)$ rad.

The efficient collection of pions emanating from a target at large angles and with a large momentum spread requires metallic lenses similar to those used for antiproton collection.[7] Such lenses are cylindrical conductors carrying a high pulsed current to create an asimuthal magnetic field providing strong linear focussing. Lithium is particularly suitable because it has the least nuclear absorption of any metal. To insure a uniform current distribution and linear focusing field in a metallic lens, the minimum pulse length is chosen to make the skin depth δ comparable to the lens radius, a. Studies of antiproton yields suggest that the lens collection efficiency at the time of maximum field linearity reaches 95% for $\delta/a \simeq 0.4$ and increases slowly for larger ratios. The lens collection efficiency is approximated by unity in the remaining discussion.

Pion Decay to Muons

The decay of charged pions to muons involves a two-body final state. The energy spectra of the muon and neutrino are both uniform in the laboratory reference frame with bounds $E_{\pm} = \gamma_\pi(E^* \pm \beta_\pi cp^*)$, where the decay momentum in the pion rest frame is $p^* = (m_\pi^2 - m_\mu^2)c/2m_\pi$. For pion energies greater than a GeV, β_π and β_μ are nearly one in the laboratory frame. The muon momentum spectrum is then essentially uniform with upper and lower bounds p_π and χp_π respectively, where $\chi \equiv (m_\mu/m_\pi)^2 \simeq 0.57$.

For a pion beam with a momentum spread $\pm\epsilon \equiv \pm\Delta p/p_\pi$ and $\epsilon \leq (1-\chi)/(1+\chi)$, the muon decay spectrum is illustrated in Figure 3. The muon fractions corresponding to the areas A,B and C in the figure are given by

$$A = \epsilon\chi/(1-\chi) = 1.3\,\epsilon \tag{1}$$

$$B = 1 - \epsilon(1+\chi)/(1-\chi) = 1 - 3.6\,\epsilon \tag{2}$$

$$C = \epsilon/(1-\chi) = 2.3\,\epsilon. \tag{3}$$

The large momentum spread of the resulting muon beam will limit the decay efficiency $\eta_{\pi\mu}$ for obtaining muons from pions.

If the pion decay occurs in a transport line of length approximately equal to the pion decay length ($53.6\,p_\pi(GeV/c)$ meters) and momentum acceptance equal to the pion momentum spread, then the decay efficiency $\eta_{\pi\mu} \simeq 0.63\,C = 1.4\,\Delta p/p_\pi$. The transport line is assumed to have an aperture adequate to transport all of the resulting muon beam with its increased emittance. The maximum muon angle from pion decay is $\theta_+ \simeq 0.282/\gamma_\pi$ radians. If $\bar\beta$ is the average betatron focusing function of the line, then the maximum increase in the invariant emittance from the decay of pions to muons is $\Delta\epsilon_N(95\%) \simeq \gamma_\pi\bar\beta\theta_+^2/2$. For example if $p_\mu \simeq p_\pi = 10$ GeV/c and $\bar\beta = 5$ meters, then $\Delta\epsilon_N(95\%) \simeq 0.28$ cm rad.

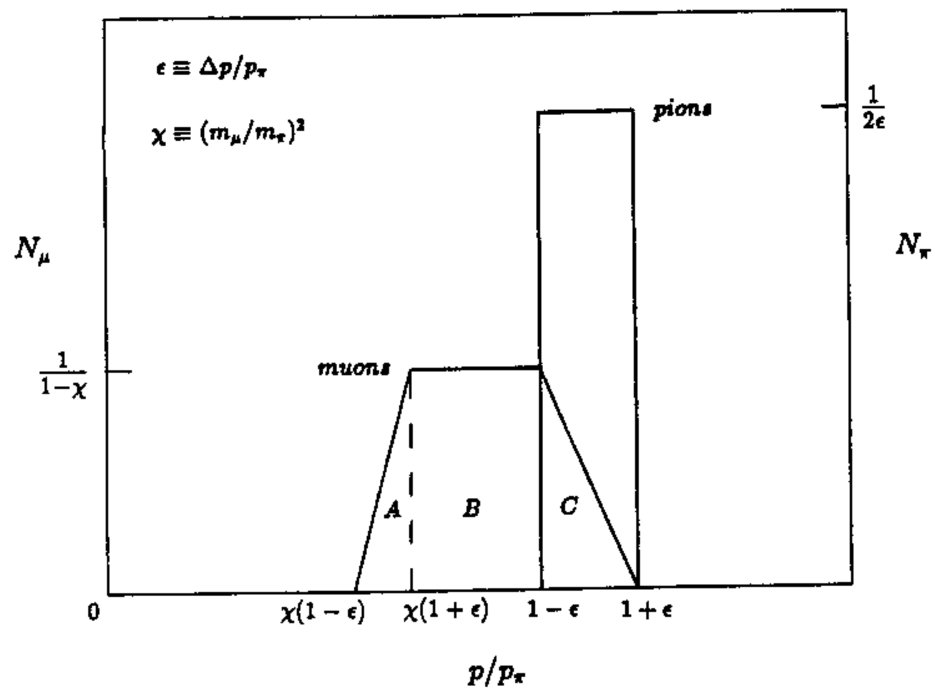

Figure 3: Normalized muon decay spectrum (left) resulting from a uniform pion spectrum (right) when β_π is nearly one. The muon fractions A,B and C are defined in Eqns. (1)-(3).

Particle Acceleration and Survival

Elementary considerations suggest that the relationship of particle lifetime and acceleration rate is central to the discussion of a rapid-cycling muon facility and is independent of the facility's details. The decay of accelerated particles is described by the equation $dN/N = -dt/\gamma\tau_o = -dz/c\tau_o(\gamma_i + zd\gamma/dz)$ where $\gamma = E/mc^2$, τ_o is the particle lifetime in the rest frame, and $d\gamma/dz$ is the acceleration gradient.

The survival ratio $\eta_a = N_f/N_i$ of unstable particles accelerated from γ_i to γ_f is

$$\eta_a = (\gamma_i/\gamma_f)^{1/(c\tau_o\, d\gamma/dz)}. \tag{4}$$

The accelerated particle survival in Eqn. (4) as a function of γ_f/γ_i is illustrated in Figure 4 for different normalized acceleration gradients. The normalized acceleration gradient $d\gamma/d(z/c\tau_o)$ is simply the energy gain in units of the rest mass per unit decay length of the particle.

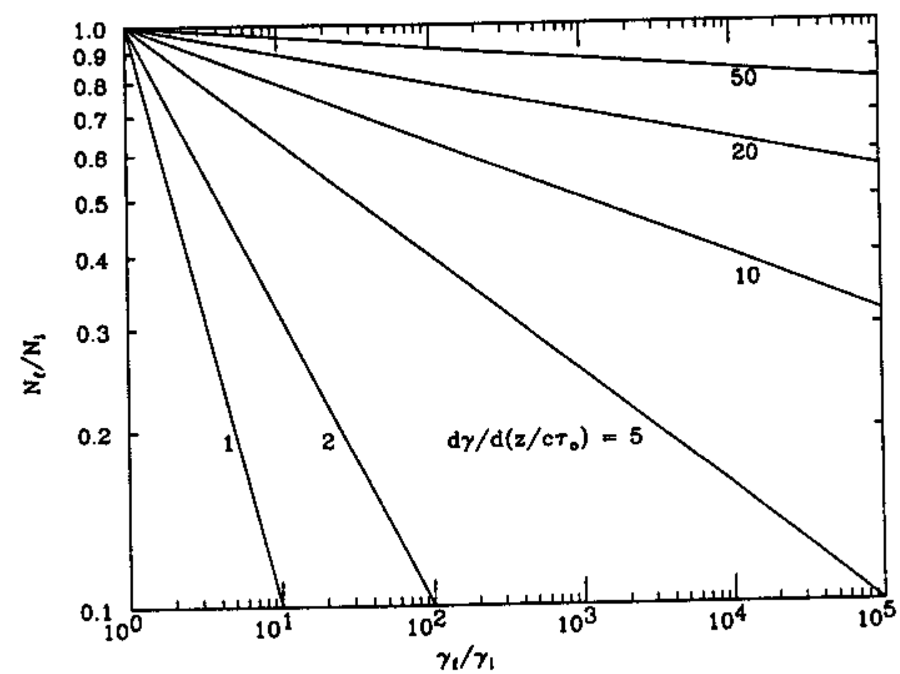

Figure 4: Particle survival as a function of energy ratio for different acceleration gradients.

For muons the range of gradients illustrated in Figure 4 is 0.15 MV/m to 7.5 MV/m. If the initial muon energy is 10 GeV, then acceleration gradients of 1 to 2 MV/m are adequate to insure a particle survival $\eta_a \simeq 0.5$ for final energies between 1 TeV and 10^3 TeV. Muon survival rapidly decreases for lower gradients so this is probably the minimum gradient acceptable for acceleration in a rapid cycling synchrotron complex. Acceleration gradients in a linac are commonly an order of magnitude higher (10 to 20 MV/m), and the muon survival would be greater than 90%.

Conclusion

Muon production, emittance cooling and acceleration are inter-related processes which ultimately must be optimized to deliver an adequate supply of low-emittance muons for a high-energy physics experiment. In the present study the approximate survival ratio of high-energy muons to initial protons is the product of the various efficiencies

$$N_\mu/N_p = \eta_{p\pi}\,\eta_t\,\eta_{\pi\mu}\,\eta_c\,\eta_a. \tag{5}$$

The pion production energy will be taken as 10 GeV (200 GeV protons) since anti-proton production near this energy is a well developed technology. The pion momentum spread is assumed to be $\pm\Delta p/p_\pi = \pm 0.05$. From Figure 2 then $\eta_{p\pi} \simeq 0.07$. The target efficiency is $\eta_t = 0.4$. If the pion decay occurs in a transport line of length equal to the pion decay length (536 meters), then $\eta_{\pi\mu} = 0.07$. The ratio of muons surviving emittance cooling will be taken as $\eta_c = 0.5$. If the acceleration to the final energy ($\leq 10^3$ TeV) occurs in a synchrotron complex, then the acceleration survival ratio is assumed to be $\eta_a = 0.5$, and from Eqn. (5) $N_\mu/N_p \simeq 5 \times 10^{-4}$. For acceleration in a linac, η_a is nearly one, and $N_\mu/N_p \simeq 10^{-3}$.

Between 10^3 and 2×10^3 protons are needed in the initial meson factory for every muon required in the high-energy collider assuming the captured pion momentum spread is ±5%. Since the ratio N_μ/N_p is approximately proportional to $(\Delta p/p_\pi)^2$ because of $\eta_{p\pi}$ and $\eta_{\pi\mu}$, there can be a significant improvement in muon production if larger momentum spreads can be utilised.

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